Extending PV to Multiple Periods

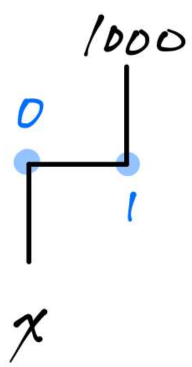
Fundamental intuition about PV:

PV of payment *F* at time *T*:

Size of bank deposit *needed at 0* to *have F at T*

Example:

Payment $1000
Period 1
$r = 10\%$

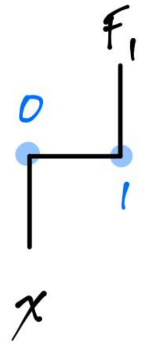


Want balance at 1 to be $1000:

$$x(1 + r) = x * 1.1 = 1000$$

$$x = \frac{1000}{1.1} = 909.09$$

Generalizing to other possible payments and interest rates:

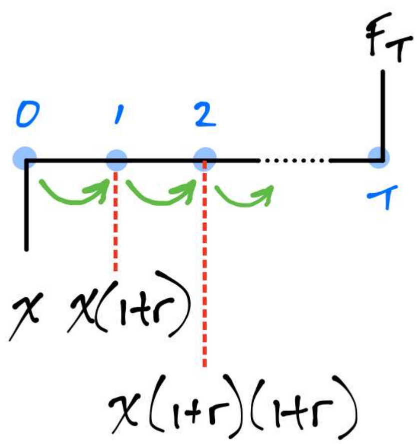


Want balance at 1 to be F_1:

$$x(1 + r) = F_1$$

$$x = \frac{F_1}{1 + r}$$

Generalizing to more periods:



Period	Balance
0	x
1	$\mathbf{x(1+r)}$
2	$[\,\mathbf{x(1+r)}\,](1+r) = x(1+r)^2$
3	$x(1+r)^3$

…	…
T	$x(1+r)^T$

Want balance at T to be F_T:

$$x(1+r)^T = F_T$$

$$x = \frac{F_T}{(1+r)^T}$$

For clarity, rename x to PV:

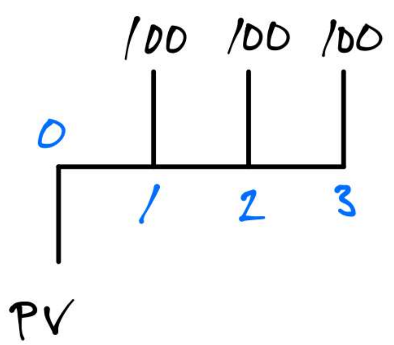
$$PV = \frac{F_T}{(1+r)^T}$$

⚠ PV formula 1

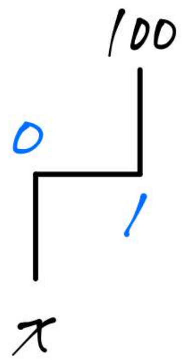
Extending to streams of multiple payments:

Example:

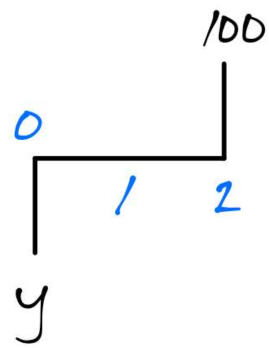
Want $100 in 1, 2 and 3
$r = 10\%$



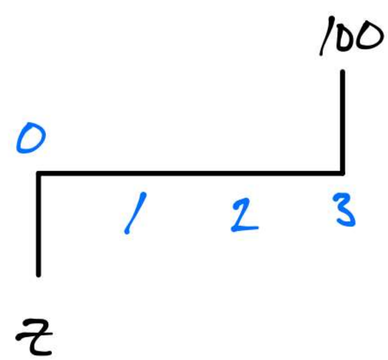
Could use 3 accounts:



$$x = \frac{100}{1.1} = 90.91$$



$$y = \frac{100}{1.1^2} = 82.64$$



$$z = \frac{100}{1.1^3} = 75.13$$

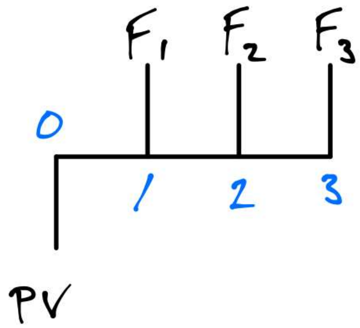
Total deposit needed:

$$PV = x + y + z$$

$$PV = \frac{100}{1.1} + \frac{100}{1.1^2} + \frac{100}{1.1^3}$$

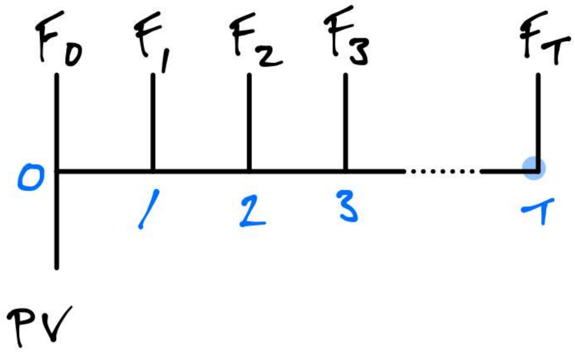
$$PV = 248.68$$

Generalizing to any three payments F_1, F_2, and F_3:



$$PV = \frac{F_1}{(1+r)^1} + \frac{F_2}{(1+r)^2} + \frac{F_3}{(1+r)^3}$$

Generalizing to any finite stream with payments from 0 to T:



$$PV = \frac{F_0}{(1+r)^0} + \frac{F_1}{1+r} + \frac{F_2}{(1+r)^2} + \frac{F_3}{(1+r)^3} + \cdots + \frac{F_T}{(1+r)^T}$$

$$PV = \sum_{t=0}^{T} \frac{F_t}{(1+r)^t}$$

⚠ PV formula 2