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Rules of Differentiation

1. Constants

If $f(x) = k$, where k is a constant, then $\frac{df}{dx} = 0$

Example: $f(x) = 3 \quad \rightarrow \quad \frac{df}{dx} = 0$

2. Powers

If $f(x) = x^n$, then $\frac{df}{dx} = nx^{n-1}$

Example: $f(x) = x^4 \quad \rightarrow \quad \frac{df}{dx} = 4x^3$

Example: $f(x) = x \quad \rightarrow \quad \frac{df}{dx} = 1$

3. Sums and Differences

If $f(x) = g(x) \pm h(x)$, then $\frac{df}{dx} = \frac{dg}{dx} \pm \frac{dh}{dx}$

Example: $f(x) = 3x^4 - 32x \quad \rightarrow \quad \frac{df}{dx} = 12x^3 - 32$

4. Products

If $f(x) = g(x)h(x)$, then $\frac{df}{dx} = \frac{dg}{dx}h + g\frac{dh}{dx}$

Example: $f(x) = x^2(x-1) \quad \rightarrow \quad \frac{df}{dx} = 2x(x-1) + x^2$

5. Quotients

If $f(x) = \frac{g(x)}{h(x)}$, then $\frac{df}{dx} = \frac{1}{h^2}\left(\frac{dg}{dx}h - g\frac{dh}{dx}\right)$

Example: $f(x) = \frac{x^2}{(7+x)} \quad \rightarrow \quad \frac{df}{dx} = \frac{1}{(7+x)^2}\left(2x(7+x) - x^2\right)$

6. Logs and Exponentials

If $f(x) = \ln(x)$, then $\frac{df}{dx} = \frac{1}{x}$

If $f(x) = e^x$, then $\frac{df}{dx} = e^x$

If $f(x) = e^{g(x)}$, then $\frac{df}{dx} = \frac{dg}{dx}e^{g(x)}$

Example: $f(x) = e^{x^2} \quad \rightarrow \quad \frac{df}{dx} = 2xe^{x^2}$

7. The Chain Rule

If $z = f(y)$ and $y = g(x)$, then $\frac{dz}{dx} = \frac{df}{dy}\frac{dy}{dx}$

Example: $z = y^2, y = (x^2 + 3x) \quad \rightarrow \quad \frac{dz}{dx} = 2\left(x^2 + 3x\right)(2x + 3)$