## Exam 2a, Fall 2004

Notes on Solution

## Part 1

Utility Function: $\quad U=X^{\wedge} a * Y^{\wedge}(1-a)$
Parameter a:
0.2

Demand for $X$ : $\quad X=\left(0.2^{*} M\right) / P x$
Demand for $Y$ : $\quad Y=\left(0.8^{*} M\right) / P y$
Expenditure Funct: $\quad \mathrm{M}=\mathrm{U}^{*}(\mathrm{Px} / 0.2)^{\wedge} 0.2$ * $(\mathrm{Py} / 0.8)^{\wedge} 0.8$

## Question 1

| $M$ | 2000 |
| :--- | ---: |
| $P x$ | 2 |
| $P y$ | 5 |


| $X$ | 200 |
| :--- | ---: |
| $Y$ | 320 |
| $U$ | 291.2903 |

## Question 1b

| Px | 4 |
| :--- | :---: |
| Py | 5 |
| X | 100 |
| Y | 320 |
| M3 | $M=291.290{ }^{*}(4 / 0.2)^{\wedge} 0.2^{*}(5 / 0.8)^{\wedge} 0.8$ |
| M3 | 2297 |
| CV | 297 |
| Revenue: | 200 |

## Question 2a



| Years: | 15 |  |
| :--- | ---: | ---: |
| Annual cost: | 28 | 420 |
| Interest rate: | $5 \%$ |  |

PV at 0 of 28 million per year forever: 560
PV at 15 of 28 million per year, years 16+ 560
PV at 0 of 28 million per year, years 16+ 269
PV cost of payments 1-15: 291

## Question 2b

```
B million
PV at 15 of benefits, years 16+ }40
PV at O of benefits, years 16+ 192
NPV = PV(B) - PV(C) -98
```

Does not make sense to clean up the lake; could deliver the benefits at a lower cost.

PV at 15 of benefits, years 16+
PV at 0 of benefits, years 16+
B/r

Benefits must be at least large enough to make the following hold:
$(B / r) /(1+r)^{\wedge} 15 \geq 291$
$B \geq r$ * $(1+r)^{\wedge} 15$ * 291
$B \geq \quad 30.20999$ million

Check:
$B / r \quad 604$
PV of B/r 291

## Question 3a

The diagram below shows the two cash flows associated with the project. The top flow is for the bond used to finance the project: it generates $\$ 1 \mathrm{~B}$ immediately but then obligates the city to raise $\$ 50 \mathrm{M}$ in interest per year. Since the CV of a $\$ 1$ tax is $\$ 1.20$, raising $\$ 50 \mathrm{M}$ costs the city's citizens $\$ 60 \mathrm{M}$ in CV. Hence, the overall cost of each interest payment is really $\$ 60 \mathrm{M}$.


Construction cost at 0 :
Annual benefits when complete:
Year benefits begin:
Interest rate:
PV of bond:
CV per year: 60
PV of payments forever: 1200
Revenue raised at 0: 1000
Net value (revenue-costs): -200
PV of road:
$\mathrm{PV}(\mathrm{B})$ at 102000
$\mathrm{PV}(\mathrm{B})$ at $0 \quad 1228$
Cost at 0: 1000
Net value (benefit-cost): 228
Overall:
PV of bond: -200
PV of road: 228
Net PV overall: 28

